Direct Parametrization of Invariant Manifolds for Model Order Reduction of Large Dimensional Finite Element Systems

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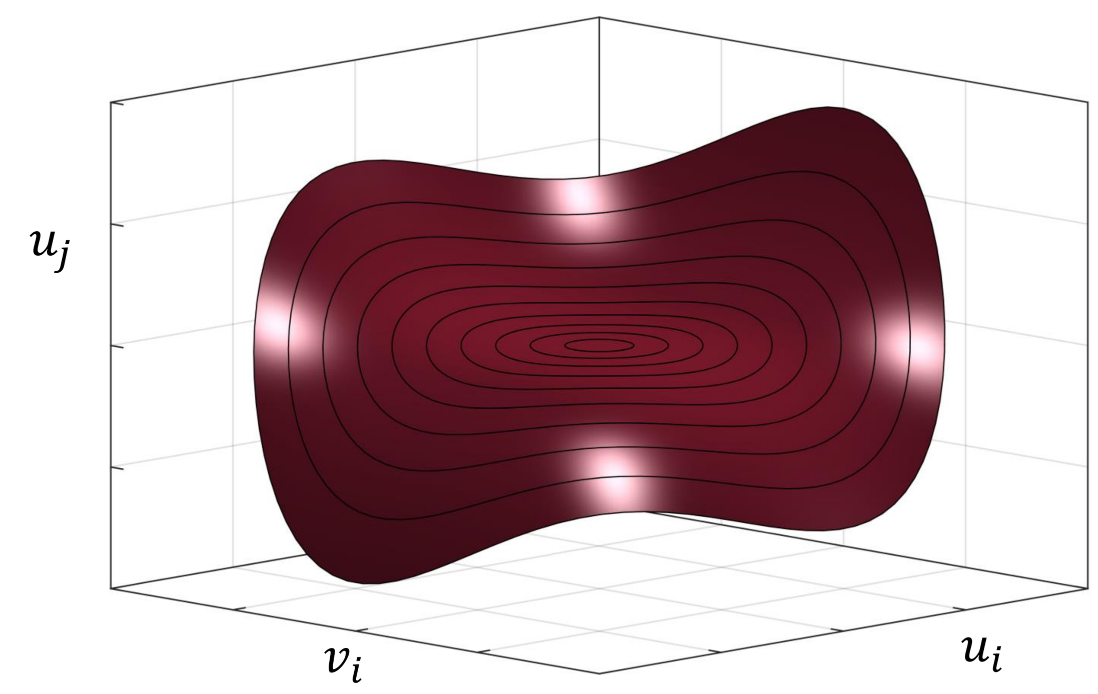
1 Invariant manifolds in structural mechanics: a simplified explanation

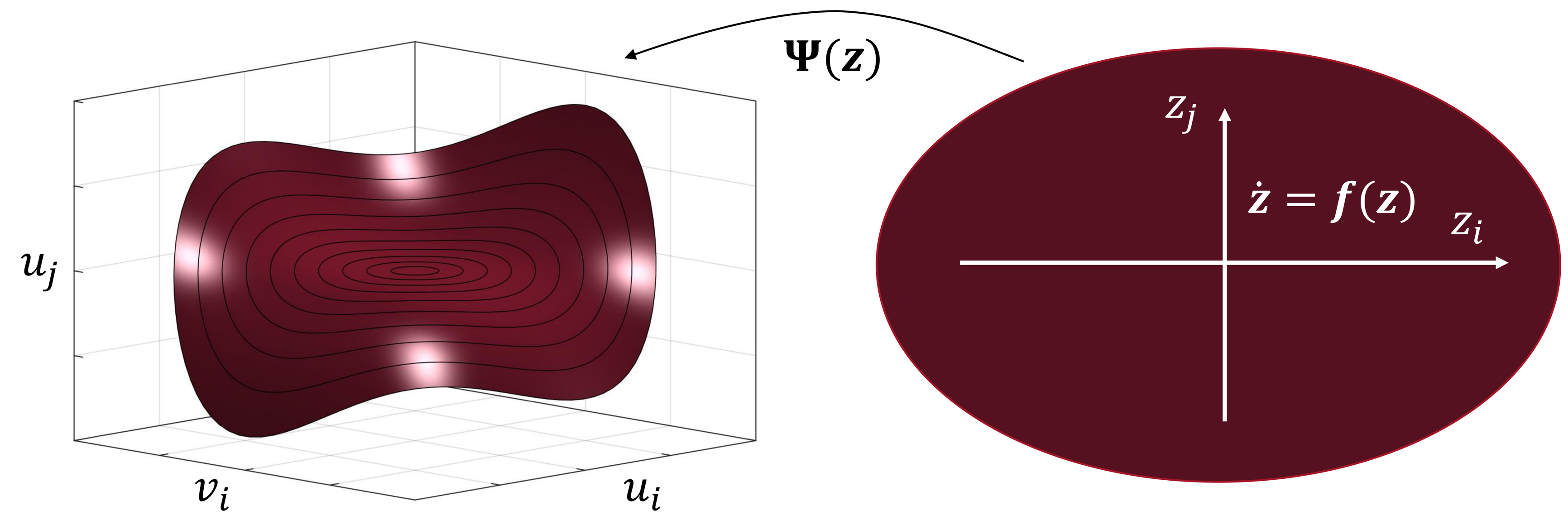
For each eigenmode of a mechanical system, it is possible to identify in phase space at least an invariant manifold tangent at the origin to the plane spanned by the linear mode [1]. This manifold is of dimension two in absence of internal resonances, and it is a curved hypersurface in presence of geometric nonlinearities. Intuitively, these subspaces can be thought as the **extension of the linear mode concept to the large amplitude regime**.



u - modal displacement v - modal velocity

2 What is a direct parametrization?

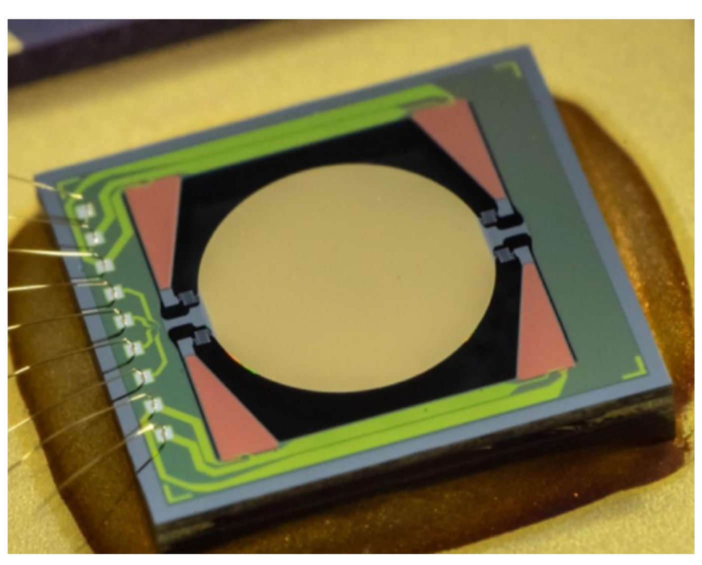
A **parametrization** is the derivation of parametric function $\boldsymbol{\Psi}(\boldsymbol{z})$ of a manifold. The number of variables $\boldsymbol{z}$ used to describe the manifold is proportional to its dimension. We also need to derive the **dynamics** of the system onto the manifold $\boldsymbol{f}(\boldsymbol{z})$ [2].

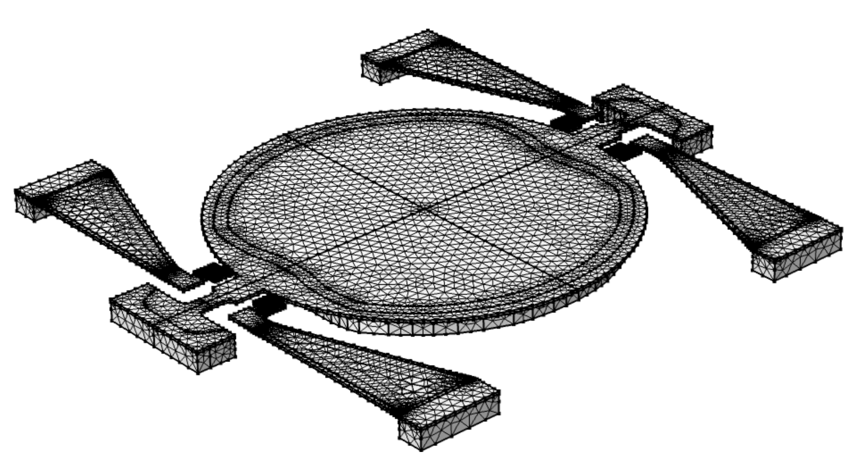


Once the dynamic of the system is known, it is possible to solve the dynamic equation on the manifold. Then, the system mapped back to physical coordinates using $\boldsymbol{\Psi}(\boldsymbol{z})$, hence retrieving the solution of the full-order model. A parametrization is **direct** if it does not require the knowledge of the full eigenspectrum of the linearized mechanical system [3-4].

3 What can we use it for?

Large dimensional finite element models are too computational demanding for designing mechanical components as Micro-Electro-Mechanical Systems (MEMS) structures excited at resonance as scanning micromirrors [5] or resonators.

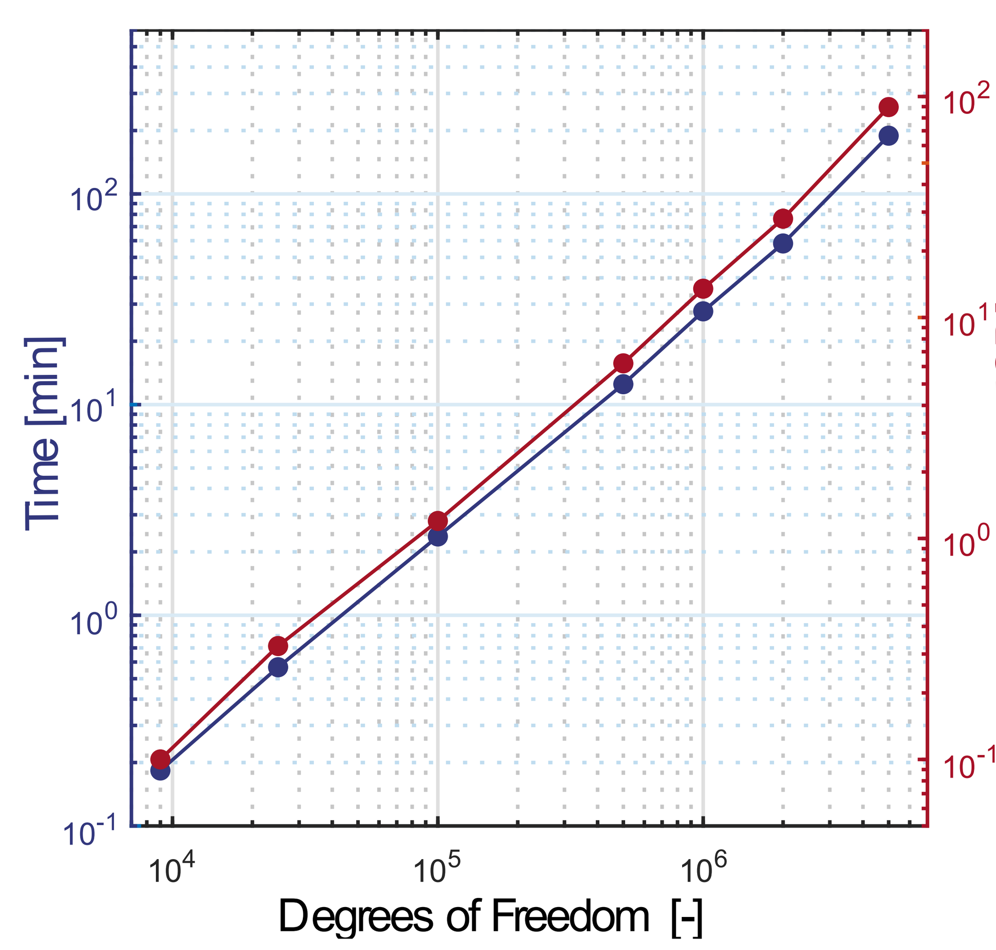




Degrees of freedom number ≈ 1M

We can parametrize the system along the invariant manifold associated to the eigenmode of the actuation mode (drive mode) and solve the reduced dynamics instead of the full order model, i.e. we exploit the direct parametrization method as a **model order reduction** technique.

4 Efficiency is its key property

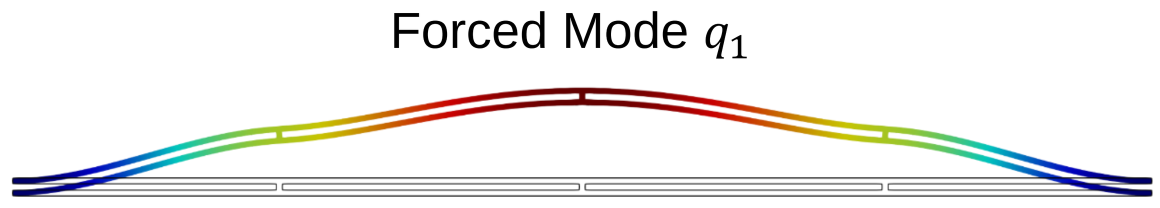


Time and memory (RAM) required to derive a reduced model as a function of the original model size. Data are reported for a parametrization along a two-dimensional manifold, that is when no internal resonances are observed.
Data obtained on a desktop workstation with CPU AMD Ryzen 5950X and 128GB RAM.

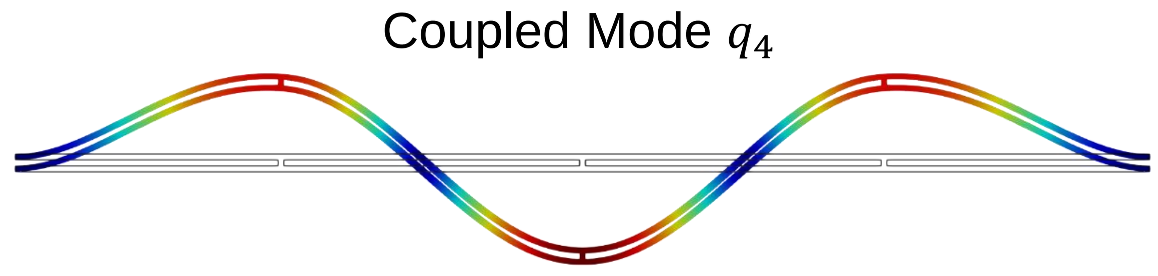
5 Example application

We applied the method to compute the steady dynamic response of a force MEMS resonators exhibiting internal resonance. The two involved eigenmodes are depicted below.

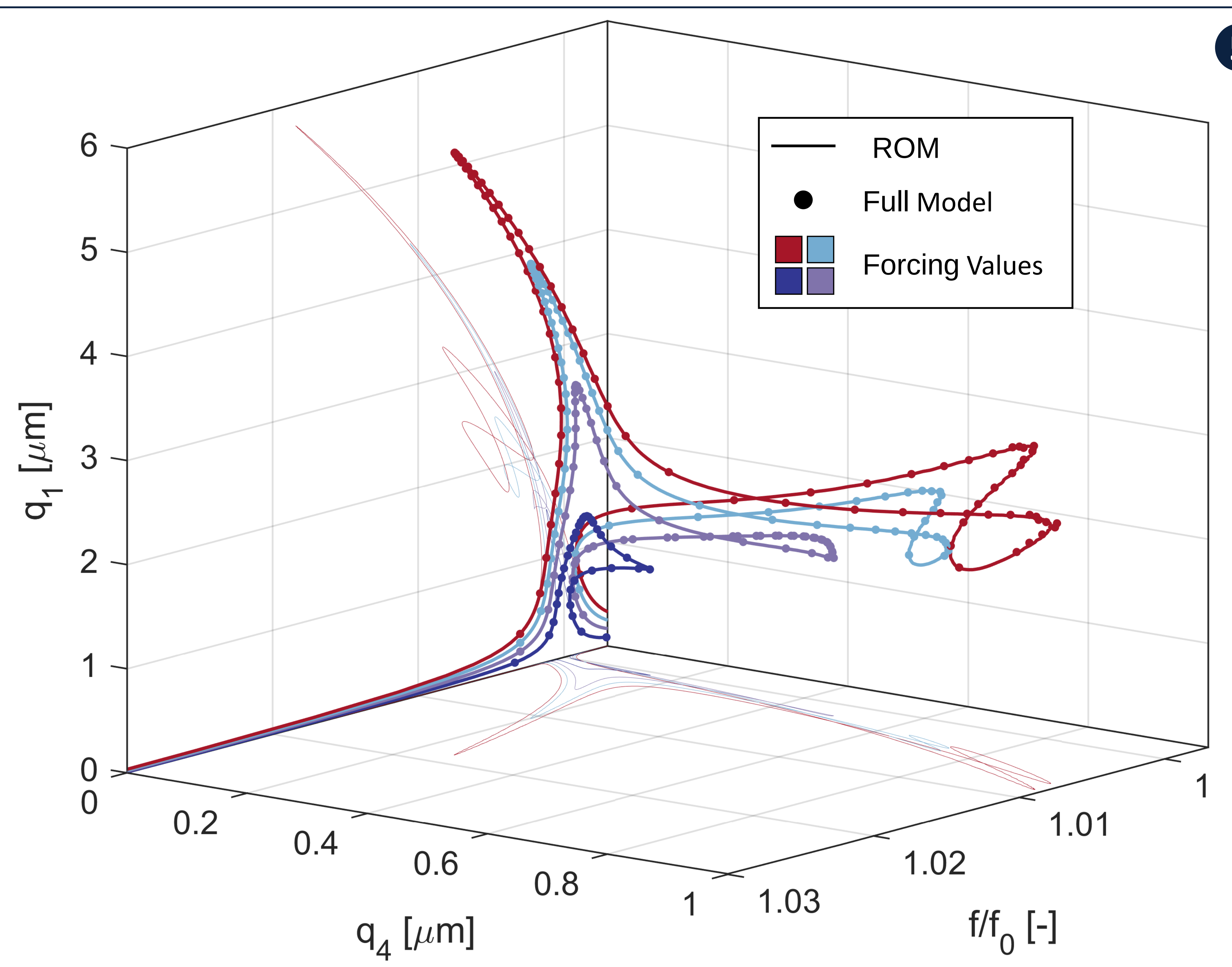
Forced Mode q_1



Coupled Mode q_4



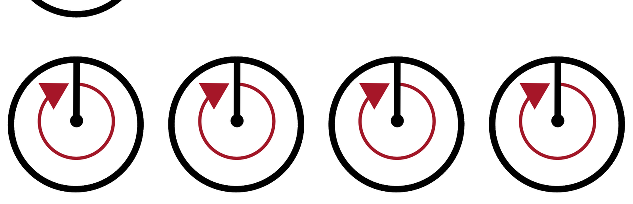
On the right we compare the predicted response of the presented approach (Reduced Order Model - ROM) with full order numerical solutions computed with the harmonic balance finite element method.



Computational Performance

ROM 10 minutes

Full Model 4 days



Foreseen publication of a high-performance implementation of the method in **Julia**

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