Multistep Deep Learning Reduced Order Models for Geometric Nonlinearities in MEMS

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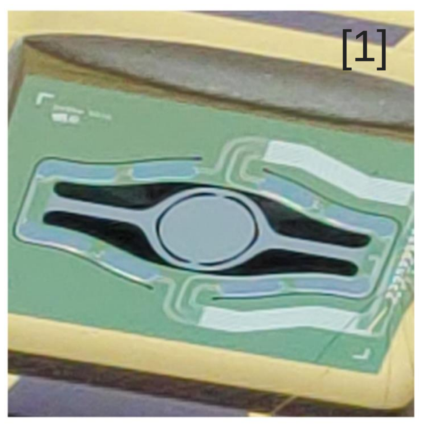
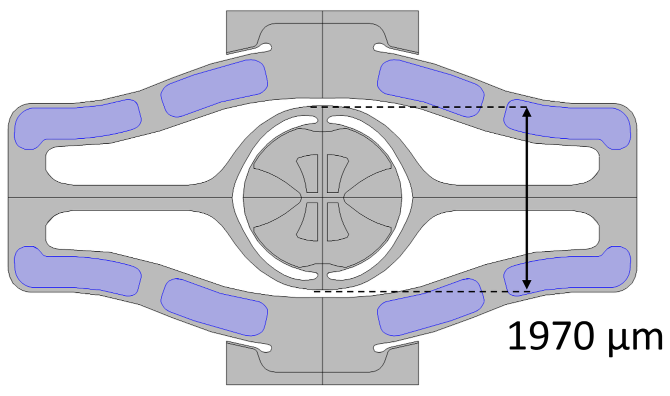
1 Target MEMS device

M icro
E lectro
M echanical
S ystems





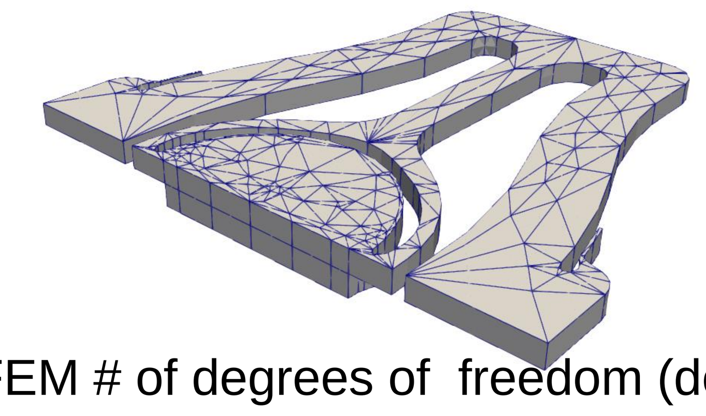
Micromirrors Applications

- The frequency of the torsional mode is 29271 Hz
- The quality factor has been set to Q = 1000
- Only geometric nonlinearities are considered

Extremely efficient reduced order model able to span the **main design parameters**

2 Finite Element Model

- The device geometry is discretised and solved with Finite Element Method (FEM)
- Harmonic balance method is used to create reference Frequency response Functions (FRF)



FEM # of degrees of freedom (dofs) N_h =9732

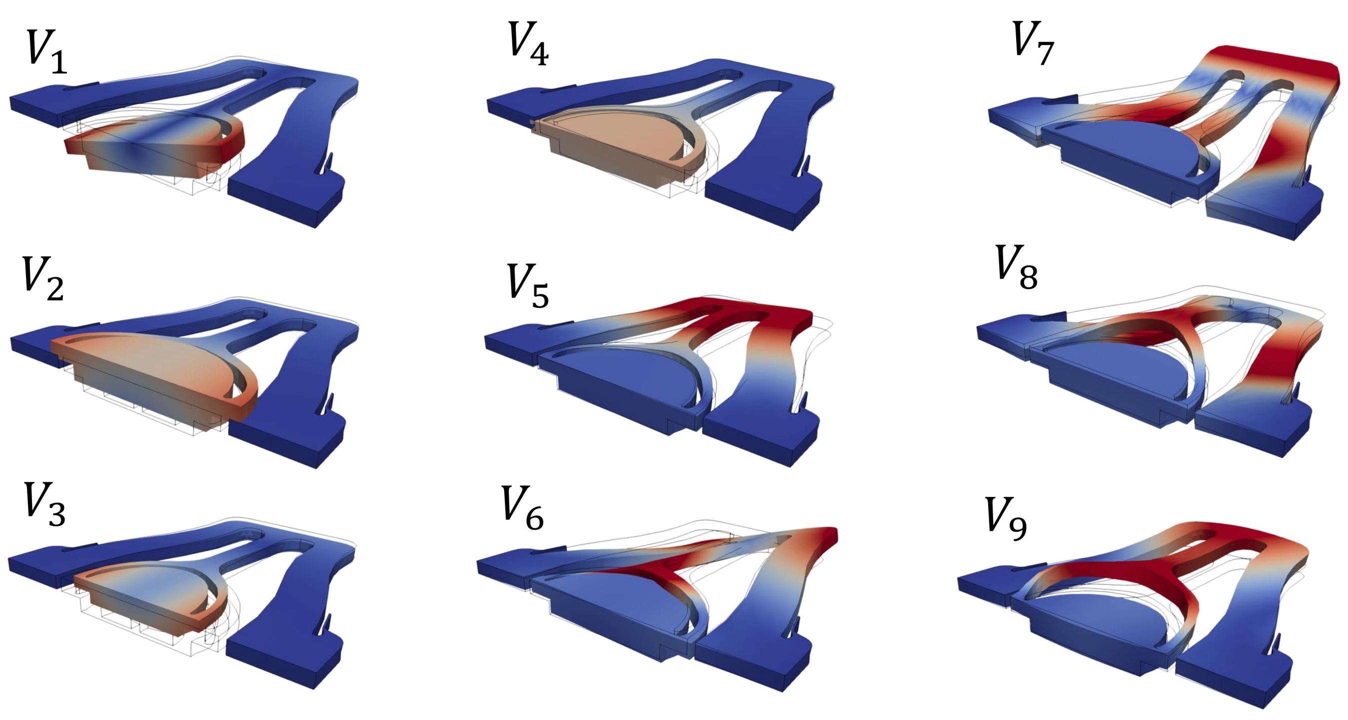
$$\mathbf{M}\,\ddot{\mathbf{D}} + \mathbf{C}\,\dot{\mathbf{D}} + \mathbf{KD} + \mathbf{G}(\mathbf{D},\mathbf{D}) + \mathbf{H}(\mathbf{D},\mathbf{D},\mathbf{D}) = \mathbf{F}(\mathbf{D},\beta,\omega,t)$$

$\mathbf{D}$ nodal displacement vector
$\mathbf{F}$ nodal force vector
$\mathbf{M}$ mass matrix
$\mathbf{C}$ Rayleigh damping Matrix
$\mathbf{K}$ Stiffness matrix
$\mathbf{G}$ vector related to 2° order monomial
H vector related to 3° order monomial
β load multiplier
ω external forcing frequency
t time
$\mathbf{F} = \beta \mathbf{M} \boldsymbol{\phi}_3 cos(\omega t)$

High Fidelity Snapshots

- Solutions of the FEM for certain t, β, ω
- 2000 snapshots collected – 40 frequencies - $\beta = 2{,}5\mu N$

3 Proper Orthogonal Modes

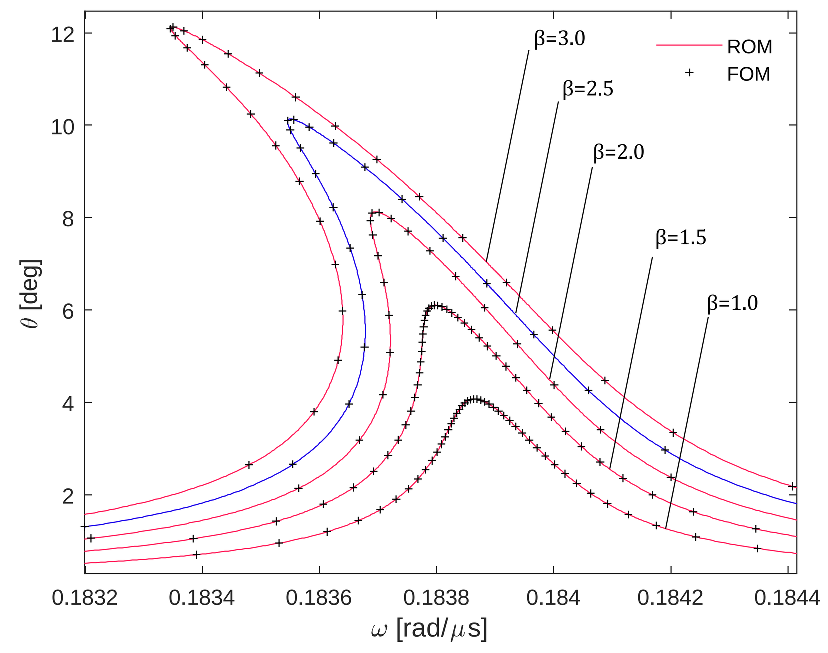


- Singular Value Decomposition is used to process the snapshots
- As results we get $\mathbf{V}$ Proper Orthogonal Modes (POMs) matrix used in the Reduced Oder Model (ROM)
- 9 bases are kept in the model

4 POD-Galerkin Exact projection of Geometric nonlinearities [5]

$$\mathbf{M}^{\mathrm{POD}}\ddot{\mathbf{Q}} + \mathbf{C}^{\mathrm{POD}}\dot{\mathbf{Q}} + \mathbf{K}^{\mathrm{POD}}\mathbf{Q} + \mathbf{G}^{\mathrm{POD}}(Q,Q) + \mathbf{H}^{\mathrm{POD}}(Q,Q,Q) = \mathbf{F}^{\mathrm{POD}}(Q,\beta,\omega,t)$$

- Proper Orthogonal decomposition (POM) generates a ROM by projecting the FEM the POMs subspace
- Since only polynomial nonlinearities are involved all the operators are projected, thus we do not need the FEM system to solve the ROM



Q= POD subspace coordinate vector
N_{POD}= dimension of the POD-ROM=9
$\mathbf{D}=\sum_i^{N_{POD}} Q_i V_i$
$\mathbf{M}^{\mathrm{POD}} = \mathbf{V}^T M \mathbf{V}$

$\mathbf{C}^{\mathrm{POD}} = \mathbf{V}^T\mathbf{C}\mathbf{V}$
$\mathbf{K}^{\mathrm{POD}} = \mathbf{V}^T\mathbf{K}\,\mathbf{V}$
$\mathbf{F}^{\mathrm{POD}} = \mathbf{V}^T\mathbf{F}$
$G_i^{\mathrm{POD}} = g_{ijk}^{\mathrm{POD}} Q_j Q_k$

$H_i^{\mathrm{POD}} = h_{ijkl}^{\mathrm{POD}} Q_j Q_k Q_l$
$g_{ijk}^{\mathrm{POD}} = \mathrm{G_i}(\mathrm{V}_j, V_k)$
$h_{ijkl}^{\mathrm{POD}} = H_i(\mathrm{V}_j, V_k, V_l)$

POD- ROM Snapshots

- Solutions given by the POD-ROM for t, β, ω
- 17 500 training snapshots – 175 frequencies on 5 load multiplier values β = 1, 1.5, 2, 2.5, 3
- 17 500 verify snapshots – 175 frequencies on 5 load multiplier values β = 1, 1.5, 2, 2.5, 3

5 POD-DLROM – offline training [6-7]

POD-ROM Snaphots $\mathbf{Q}(t,\omega,\beta)$ → $\frac{\omega_h}{2}\left|\left|\mathbf{Q}(t,\omega,\beta) - \tilde{\mathbf{Q}}(t,\omega,\beta)\right|\right|^2$

Encoder → $\tilde{\mathbf{U}}_{NN}(t,\omega,\beta)$ → $\frac{1-\omega_h}{2}\left|\left|\mathbf{U}_{NN}(t,\omega,\beta) - \tilde{\mathbf{U}}_{NN}(t,\omega,\beta)\right|\right|^2$

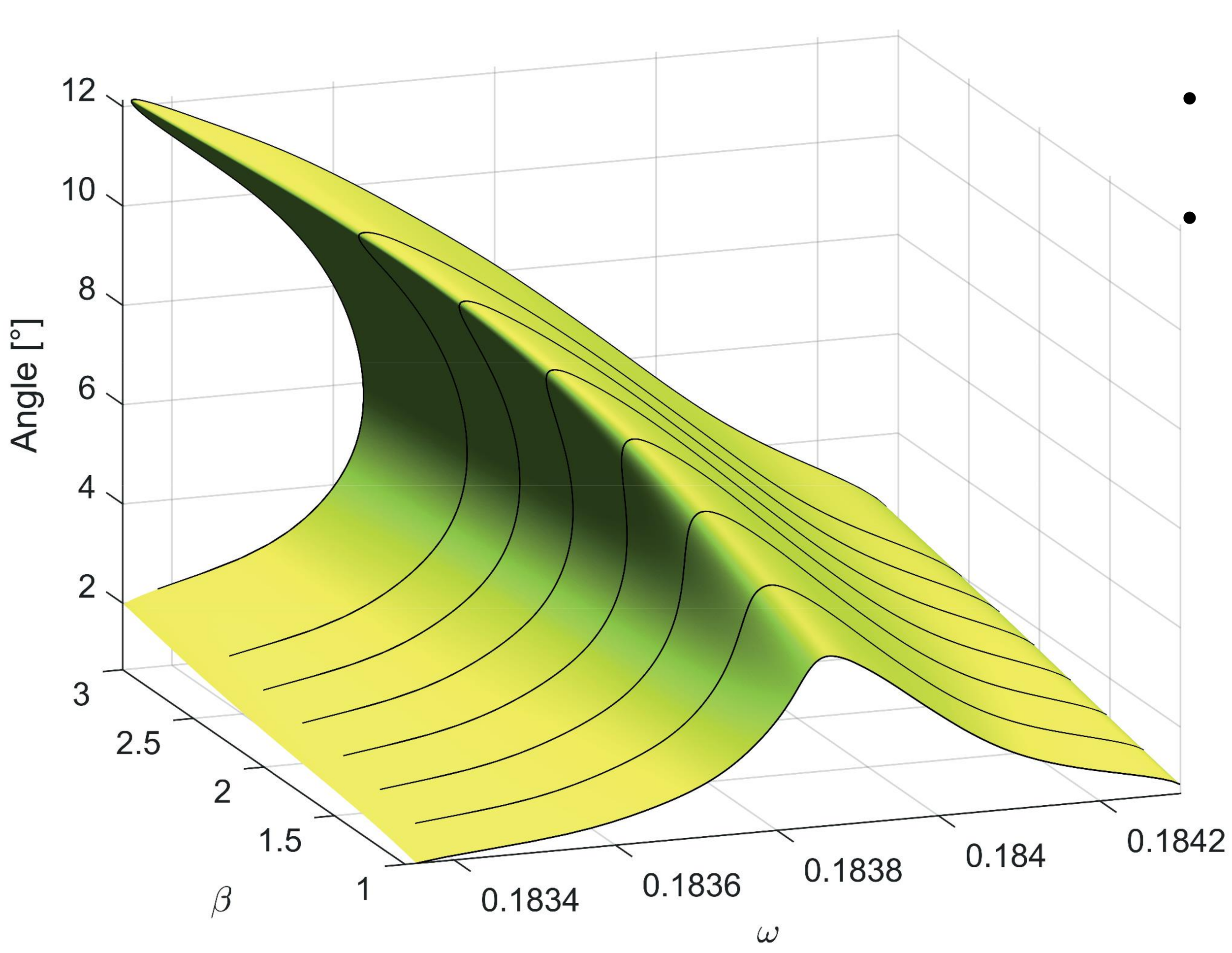
(t,ω,β) → Deep Feedforward NN (DFNN) → $\mathbf{U}_{NN}(t,\omega,\beta)$ → Decoder → $\tilde{\mathbf{Q}}(t,\omega,\beta)$

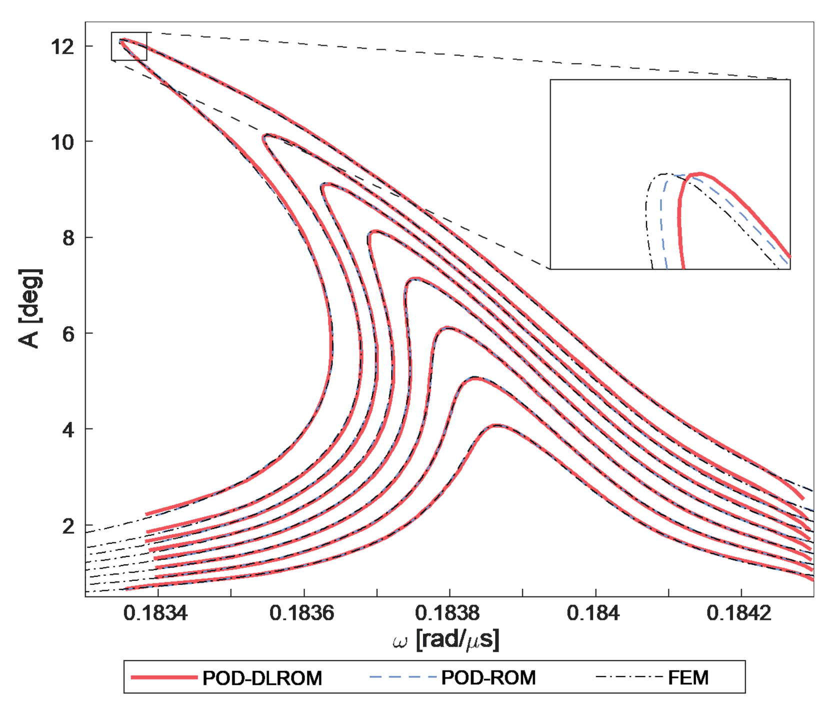
- The POD are used to train the POD-DLROM Neural Network (NN)
- The encoder nonlinearly further reduces the system to an intrinsic coordinate $\boldsymbol{U}_{NN}$ (t,ω,β)
- The DFNN for given (t,ω,β) is trained to give the same $\boldsymbol{U}_{NN}$ (t,ω,β)
- The decoder nonlinearly reconstructs from $\boldsymbol{U}_{NN}$ (t,ω,β) to $\boldsymbol{Q}(t,\omega,\beta)$

6 Results of multistep-POD-DLROM [8]

(t,ω,β) → DFNN → $\mathbf{U}_{NN}(t,\omega,\beta)$ → Decoder → $\mathbf{Q}(t,\omega,\beta)$ → $\mathbf{D}(t,\omega,\beta)$=$\mathbf{VQ}$

- In the online stage only the DFNN and the decoder are used
- For given (t,ω,β) the method provides a displacement field $\mathbf{D}$
- Good to excellent accuracy is reached





# instances	T_{FEM}	$T_{POD-\boldsymbol{ROM}}$	$T_{POD-\boldsymbol{DLROM}}$	$\frac{\boldsymbol{T_{FEM}}}{\boldsymbol{T_{POD-DLROM}}}$	$\frac{\boldsymbol{T_{POD}}}{\boldsymbol{T_{POD-DLROM}}}$
60 360 300	489 days	6,8 hours	12 sec	3,5 10^6	2 10^3

[1] Frangi A., Opreni A., Boni N., Fedeli P., Carminati R., Merli M., & Mendicino G. (2020). Nonlinear response of PZT-actuated resonant micromirrors. Journal of Microelectromechanical Systems, 29(6), 1421-1430.
[2] https://www.microsoft.com/en-us/hololens/
[3] https://www.bynorth.com/
[4] https://www.st.com/content/dam/AME/2019/developers-conference-2019/presentations/STDevCon19_2.4-6-Laser-Beam-Scanners-ST.pdf
[5] Gobat G., Opreni A., Fresca S., Manzoni A., Frangi A, (2021) "Reduced order modeling of nonlinear microstructures through Proper Orthogonal Decomposition", Mechanical Systems and Signal Processing, under review
[6] Fresca S., Dede L., Manzoni A. "A comprehensive deep learning-based approach to reduced order modeling of nonlinear time-dependent parametrized PDEs." Journal of Scientific Computing 87.2 (2021): 1-36.
[7] Fresca S., and Manzoni A. "POD-DL-ROM: enhancing deep learning-based reduced order models for nonlinear parametrized PDEs by proper orthogonal decomposition." arXiv preprint arXiv:2101.11845 (2021).
[8] Fresca S., Gobat G., Manzoni A., Frangi A., Deep learning-based reduced order models for Micro-Electro-Mechanical systems, keynote presentation at MMLDT 2021

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