Seminar Announcement

Beltrami Room, Building no. 5, Ground Floor, Leonardo Campus (Piazza L. da Vinci, 32)

Department of Civil and Environmental Engineering

Monday, 22nd June 2015, from 12:00 to 13:00

On the second-order homogenization of wave motion in periodic media and the sound of a chessboard

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Abstract

The goal of this study is to understand the mathematical structure and ramifications of the second-order homogenization of low-frequency wave motion in periodic solids. To this end, multiple-scales asymptotic approach is applied to the scalar wave equation in one and two spatial dimensions. In contrast to previous studies where the second-order homogenization has led to a single fourth-order derivative in the governing equation, present investigation demonstrates that such (asymptotic) approach results in a family of field equations uniting spatial, temporal, and mixed fourth-order derivatives – that jointly control the incipient wave dispersion. Given the consequent freedom in selecting the affiliated lengthscales parameters, the notion of an optimal asymptotic model is next considered in a one-dimensional setting via its ability to capture the salient features of wave propagation within the first Brillouin zone, including the onset and magnitude of the phononic band gap.

Considering the wave motion in two dimensions, on the other hand, the asymptotic analysis is first established in a general setting, exposing the constant shear modulus as a sufficient condition under which the second-order approximation of a bi-periodic solid is both isotropic and limited to even-order derivatives. On adopting a chessboard-like periodic structure as a testbed for in-depth analytical treatment, it is next shown that the Mindlin’s celebrated long-wavelength approximation of gradient elasticity is incomplete for it does not capture the anisotropic wave dispersion characterized by the “$\sin^4 \theta + \cos^4 \theta$” term — observed e.g. in chessboard and related periodic structures.

The presentation will conclude with the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficients. The analysis demonstrates that in this case there are two sources of (low-frequency) asymptotic perturbation, namely i) the bulk correction — which is described in the main
part of this talk, and ii) the boundary correction due to a compact support of the periodic structure. For completeness, the study will provide the $H^1$ and $L^2$ estimates of the error committed by the homogenized (scattered field) solution, and highlight the difficulties in characterizing the boundary correction.

**Biographical sketch**

Bojan Guzina's major areas of research are in the field of elastic wave propagation with applications to nondestructive site and material characterization. Projects accomplished include analytical modeling of seismic wave motions in a multi-layered solid, development of a rigorous boundary integral equation (BIE) method for visco-elastodynamic problems, and formulation of a BIE-based imaging algorithm for 3D seismic identification of subterranean objects. Current research interests are focused on: (i) resolution of the inverse scattering problems arising in non-intrusive site characterization and underground object detection, (ii) development of novel field testing techniques.

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Reference person: Prof. Attilio Frangi (attilio.frangi@polimi.it)